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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL TRANSLATION F-2

ON THE TEMPERATURE DISTRIBUTION BEHIND CYLINDERS IN A FLOW*

By Jakob Ackeret

If a body which is assumed to be heat-insulated is in the flow of a gas flowing uniformly parallel for a relatively large distance, differences in temperature result along the surface and in the wake. For an ideal gas flow (without separation) the steady state becomes (fig. 1):

$$\epsilon = c_p T_i + \frac{c^2}{2} = \text{Constant} = c_p T_\infty + \frac{U^2}{2} \tag{1}$$

Thus,

$$T_i - T_\infty = \frac{U^2 - c^2}{2c_p}$$

At the stagnation points S_1 and S_2 where $c = 0$,

$$T_0 = T_\infty + \frac{U^2}{2c_p}$$

would be the stagnation or tank temperature.

Therefore,

$$T_i = T_0 - \frac{c^2}{2c_p} \tag{2}$$

Actually, however, boundary layers exist which can still separate in the case of blunt forms. The theory of compressible boundary layers yields for the attached flow, thus for very slender profiles, values of wall temperature T_w which are higher than T_i. Thus a temperature recovery is present which, therefore, is largely incomplete.

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*"Über die Temperaturverteilung hinter angeströmten Zylindern." Mitt. No. 21, Inst. für Aerod., Tech. Hochschule, (Zürich), 1954, pp. 5-21. Presented at the Fall Meeting of the Swiss Physical Society at Bern on August 24, 1952. (See ZAMP, vol. 3, issue 6, 1952, p. 472.)

For laminar boundary layers on plane plates,

$$T_w = T_i + \sqrt{\sigma}\left(T_0 - T_i\right) = T_i + \sqrt{\sigma}\,\frac{c^2}{2c_p} = T_0 - \left(1 - \sqrt{\sigma}\right)\frac{c^2}{2c_p} \qquad (3)$$

where $\sigma = \frac{\mu c_p}{\lambda}$ is the so-called Prandtl number. $\left(\mu\right.$ = viscosity, c_p = specific heat for constant pressure, λ = heat conductivity$\left.\right)$.

Since for air $\sigma \cong 0.70$, there follows with $\beta = \sqrt{\sigma} = 0.83$

$$T_w = T_i + 0.83\left(T_0 - T_i\right) = T_0 - 0.17\,\frac{c^2}{2c_p} \qquad (3a)$$

It seems that this relationship applies rather satisfactorily also in those cases where a pressure drop in the flow direction exists (see below).

In the case of turbulent boundary layers, the recovery is larger. According to the concept of turbulent exchange, $\sigma = 1$ would be expected, thus complete recovery. Actually, however, there exists also a laminar sublayer which reduces the effective recovery. In practice, the following condition is found approximately

$$T_w = T_i + 0.90\left(T_0 - T_i\right) = T_0 - 0.10\,\frac{c^2}{2c_p} \qquad (4)$$

Therefore, almost complete recovery is obtained in all cases; only at points of high excess velocity is a noticeable difference $\left(T_0 - T_w\right)$ present.

In the case of blunt forms with flow separation, a new phenomenon appears: particularly low temperatures are measured on the back side (1, 2). This is remarkable because the flow velocities there certainly are small, so that, according to the steady energy equation (1), stagnation temperature should almost be reached.

Ryan[1] has shown that this "aerodynamic cooling" appears for very different cylinder forms and that profiles exist which not only do not yield any temperature recovery but rather a cooling below the free-stream

[1]Ryan, Lloyd F.: Experiments on Aerodynamic Cooling. Mitt. No. 18, Inst. für Aerod., Tech. Hochschule (Zürich), c. 1951. The figures of that paper, referred to herein with the prefix Ry, have not been included in this paper.

temperature T_∞. Figures Ry 21 and Ry 12, Ry 16, Ry 17, and Ry 18 show for circular cylinders, half bodies, and various other shapes, the temperatures at the rear point of the profile. The profile named as the first in figures Ry 12, Ry 16, Ry 17, and Ry 18 yields, for instance, at the Mach number $\frac{U}{a} = 0.6$, a recovery factor of -0.65. It would evidently be interesting to attempt an explanation of this temperature deficit.

A first qualitative hint is offered by certain observations that Ryan made. He found that the cooling effect is particularly large when the noise caused by the vortices being shed at high speeds (probably due to resonance phenomena) appears amplified. Figure Ry 13 shows a schlieren photograph of the vortices behind a circular cylinder for a state of amplified noise; figure Ry 19 shows the vortex formation behind the two-dimensional half body. Here it is remarkable that the sound waves which occur during formation and separation of the vortices can be seen - an indication, by the way, that the separation is dynamically a very effective process. Figure 2 of this paper (according to a flow photograph in water) shows that a rather regular flow exists near the cylinder.

Another of Ryan's important observations concerns the fact that a strong cooling occurs also in the case of a wing profile when separation sets in and strong individual vortices are forming. It could be assumed that it is not so much the irregular turbulence which causes the cooling but nonsteady well-defined flows during the separation and the flowing off of the vortices. In this case, it could be assumed, furthermore, that even the potential flow of the field of individual vortices being shed may yield cooling.

For a nonsteady potential flow there applies the simple theorem[2] that the total energy $\epsilon = c_p T + \frac{c^2}{2}$ has the following relationship to the velocity potential:

$$\epsilon + \frac{\partial \phi}{\partial t} = \text{Constant in the field} \tag{5}$$

If the state (outside of the wake) at a large distance is considered, it is found to be practically steady, and

$$\epsilon_\infty = c_p T_\infty + \frac{U^2}{2} \tag{6}$$

is valid. For an arbitrary point, then

[2] Ryan, l.c., p. 5. A simple derivation may be found in appendix 1, p. 8.

$$\Delta\epsilon = \epsilon - \epsilon_\infty = -\frac{\partial\phi}{\partial t} \tag{7}$$

Only the time average is measured. It results from the calculation as

$$\bar{\epsilon} = \epsilon_\infty - \frac{1}{t_1} \int_0^{t_1} \frac{\partial\phi}{\partial t}\, dt = \epsilon_\infty - \frac{\phi_{t_1} - \phi_0}{t_1} \tag{8}$$

so that only the initial and final potential values need be calculated. This is very convenient inasmuch as the motion of the vortex centers need not be known in all details.

The simple calculation for the circular cylinder is now to be performed. According to the observations, alternating individual vortices with the circulations $\pm\Gamma$ are formed, n pairs (+ -) per unit time. The flow about the cylinder then is to be composed of a cylinder flow without circulation and of a nonsteady vortex plus reflected vortex field which may be superposed. If a vortex-reflected vortex pair (fig. 3(a)) is considered, the potential at an arbitrary point P is given by

$$\phi_P = \frac{\Gamma}{2\pi} \varphi_1 + \frac{\Gamma}{2\pi} \varphi_2 = \frac{\Gamma}{2\pi} \omega \tag{9}$$

If the vortex forms at an arbitrary point near the wall, the reflected vortex is at first, very close; ω_a is therefore small. As the vortex flows away, ω becomes larger and attains finally ω_e (fig. 3(b)). For the rearmost point S_2, $\omega_a \sim 0$, $\omega_e = \pi$.

Thus,

$$\frac{\partial\phi}{\partial t} = \frac{\Gamma}{2\pi} \frac{\partial\omega}{\partial t}$$

and

$$\frac{\phi_{t_1} - \phi_0}{t_1} = \frac{\Gamma}{2\pi} \frac{\omega_{t_1} - \omega_0}{t_1}$$

for a long time t_1.

The entire ϕ-variation for a vortex being shed is, for the rearmost point,

$$\frac{\Gamma}{2\pi}\pi = \frac{\Gamma}{2} \tag{10}$$

During the long time t_1, this process occurs $n \times t_1$ times and since, as is easily seen, the negative vortices yield the same energy variation, the following relation is obtained:

$$\Delta\epsilon = -\frac{\Gamma}{2}\frac{nt_1}{t_1} 2 = -n\Gamma \tag{11}$$

whereas the variation of the stagnation temperature is equal to $-\frac{n\Gamma}{c_p}$. It is rather remarkable that the position of the point P plays an important role insofar as points P_i, which lie between the vortex paths (under the assumption that $\omega_a = 0$), yield in the entire wake the same value $\Delta\epsilon$ as has just been calculated, whereas points P_a outside of this region yield zero. Thus there would be an energy decrease in the wake region, though none outside of it (fig. 4).

Doubtlessly, this picture is somewhat too simple. It is true that every rotation starts from the boundary layer, thus from the cylinder surface; for every elementary vortex, ω_a would, therefore, be zero. However, the shedding off is not always at the same point and this exerts probably an effect as though the full circulation of the collected vortex would be present only at some distance from the cylinder. Once this assumption is made, the determination of the energy takes a more complicated course (fig. 5)[3]. Here the assumption is that the vortices have formed at $\varphi = 110^{\circ}$, at a distance 1.3R, and that they move rectilinearly rearward. (The law of motion itself does not matter.) Something of the cooling then "slips" through toward the front and even at the foremost point another slight drop would have to be expected. Besides the dead region, however, there is now a small heating particularly near the cylinder and near the vortex path.

It is interesting to see that the observations of Ryan show a quite similar behavior, although they can hardly be very reliable in this area. Figure Ry 25 shows temperature measurements in the wake where the thermoelement was corrected according to special measurements. The upper curve (corrected) shows low temperatures in the wake; in addition, however, it shows temperatures higher than the stagnation temperature - that is, the effect to be expected according to the calculation.

The temperatures on the cylinder surface can be seen from figure 6. Of course, these are at first the ideal temperatures, so that another

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[3]According to a calculation for which I am indebted to Dipl. Eng. L. Meyer.

correction is required which takes the recovery in the boundary layer into consideration.

Figure 6 (top) indicates the energy distribution for the nonsteady potential flow. The dashed straight lines correspond to a shedding of the vortices directly from the surface, the solid curve corresponds to the temperature distribution for the case where the vortices form at some distance away. The recovery in the boundary layer is plotted in figure 6 (center). It would be rather difficult to calculate it exactly since a locally accelerated and, moreover, nonsteady flow is concerned. Procedure, by way of approximation, was as follows: The recovery $\frac{\beta\bar{c}^2}{2c_p}$ was determined with $\beta = 0.83$, with the velocities $\bar{c}$ calculated from the practically steady measured pressure distribution, on the front side. On the rear side ($\varphi > 110^\circ$), the velocity $\bar{c}$ was put equal to zero for this calculation. As can be seen from appendix 2, these velocities are small; in the neighborhood of the rearmost point they are, moreover, periodic so that neglecting them is intuitively suggested.

It must first be remarked that during the test (fig. 6, bottom) the vortex field was treated as incompressible. For this reason, only measurements at moderate Mach numbers ($M = 0.35$) may be used. For the decisive product $n\Gamma$ a value

$$n\Gamma = 1.17\,\frac{U^2}{2}$$

is chosen in such a manner that the temperature of the rearmost point ($\varphi = 180^\circ$) turns out according to the measurements, since a direct determination of Γ does not seem possible in practice.

It can be seen that the variation of the curves is quite satisfactory, and it must merely be determined whether the selected $n\Gamma$ contradicts other experiments.

Ryan's measurements have been made at Reynolds numbers of the order of magnitude 100,000, whereas most vortex observations pertain to much smaller Reynolds numbers. For the Kármán street, at some distance from the cylinder, it was found[4] that:

$$n = 0.202\,\frac{U}{2 \times R}$$

$\Gamma = 3.4 \times U \times R$ (according to Kármán's theory of the vortex street)

$$n \times \Gamma = 0.69\,\frac{U^2}{2}$$

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[4]Handbuch der Experimentalphysik IV 1, (Leipzig), 1931, pp. 164-165 (Falkenhagen-Schmiedel).

It is seen that a product $n\Gamma$ larger by 70 percent is needed. It must be remarked that, with the finite velocity at the rearmost point taken into consideration, $n\Gamma$ may be somewhat smaller than had been assumed in figure 6. But it is not known whether Γ is considerably larger in the neighborhood of the cylinder than at some distance, since the convergence of positive and negative circulations is to be expected, anyway,[5] where Γ then would be smaller further to the rear. Since the maximum steady velocity before the separation is approximately 1.5U, the circulation leaving per second (corresponding to $n\Gamma$) could be at most $\frac{c^2_{max}}{2} = 2.25 \frac{U^2}{2}$; this is about twice as much as is required for explaining the cooling effect. The required $n\Gamma$ can then be regarded as lying within the range of possibility.

It is noteworthy that at the point $\varphi = 0$, another small drop should also be present. The measurements actually lie below the stagnation value.

Thus it is seen that an explanation can be obtained for the cooling on the basis of a nonsteady potential flow. It is possible to calculate other cases as well and to predict heat effects so that perhaps further experimental checks can be carried out. (See appendix 3.) Probably there exists also a connection with the more recent experiments by H. Sprenger.[6]

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[5] Heisenberg, Werner: Physikalische Zeitschrift, Jg. 23., 1922, pp. 363-366, with appendix by L. Prandtl.

[6] Sprenger, H.: Über thermische Effekte in Resonanzrohren. (Mitt. No. 21 (refer to footnote on p. 1 of this translation).)

APPENDIX 1

DERIVATION OF EQUATION 5

The equations of motion of two-dimensional frictionless compressible gas flows are

$$\rho\left\{\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right\} = -\,\frac{\partial p}{\partial x}$$

$$\rho\left\{\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\,\frac{\partial v}{\partial y}\right\} = -\,\frac{\partial p}{\partial y}$$

If there exists, moreover, a potential ϕ, it follows that

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Substituting this equation gives

$$\rho\left\{\frac{\partial u}{\partial t} + \frac{\partial\left(c^2/2\right)}{\partial x}\right\} = -\,\frac{\partial p}{\partial x}$$

$$\rho\left\{\frac{\partial v}{\partial t} + \frac{\partial\left(c^2/2\right)}{\partial y}\right\} = -\,\frac{\partial p}{\partial y}$$

Multiplication by dx and dy, respectively, and addition yields

$$\rho\left(\frac{\partial u}{\partial t}\,dx + \frac{\partial v}{\partial t}\,dy\right) + \rho\;d\left(c^2/2\right) = -dp \tag{12}$$

The first term may be written, with introduction of ϕ, as

$$\rho\;\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial x}\,dx + \frac{\partial\phi}{\partial y}\,dy\right) = \rho\;\frac{\partial}{\partial t}\,d\phi = \rho d\;\frac{\partial\phi}{\partial t} \tag{13}$$

Thus,

$$d\;\frac{\partial\phi}{\partial t} + d\left(c^2/2\right) = -\frac{dp}{\rho}$$

The isentropic relationship gives $\left(\text{with} \quad \gamma = \frac{c_p}{c_v}\right)$

$$\frac{dp}{p} = \frac{\gamma}{\gamma - 1} \frac{dT}{T}$$

and

$$\frac{dp}{\rho} = \frac{\gamma}{\gamma - 1} R \, dT = c_p \, dT$$

Thus there follows:

$$d \frac{\partial \phi}{\partial t} + d\left(c^2/2\right) = -c_p \, dT$$

and with $\epsilon = c_p T + \frac{c^2}{2}$, finally

$$\epsilon + \frac{\partial \phi}{\partial t} = \text{Constant} = \epsilon_\infty$$

This equation is valid also for spatial potential flows.

APPENDIX 2

CALCULATION OF THE VELOCITY AT THE POINT $\varphi = 180^{\circ}$

Vortex and reflected vortex yield, at the point z of the periphery of the circle (fig. 7), the complex potential

$$\chi = \frac{i\Gamma}{2\pi}\left\{\ln\left(z - a_1\right) - \ln\left(z - a_2\right)\right\} \tag{14}$$

If $a_1 = \sigma R$, since $a_1 a_2 = R^2$,

$$a_2 = \frac{R}{\sigma}$$

Furthermore,

$$z = Re^{i\psi}$$

The complex velocity is

$$\frac{d\chi}{dz} = \frac{i\Gamma}{2\pi}\frac{a_1 - a_2}{\left(z - a_1\right)\left(z - a_2\right)} = \frac{i\Gamma}{2\pi R}\frac{\sigma^2 - 1}{\sigma}\frac{1}{\left(e^{i\psi} - \sigma\right)\left(e^{i\psi} - 1/\sigma\right)} \tag{15}$$

Only the magnitude of the velocity is important:

$$\left|\frac{d\chi}{dz}\right| = c$$

After some calculation, it is found to be

$$\frac{c}{\Gamma/2\pi R} = \frac{\sigma^2 - 1}{\sigma^2 + 1 - 2\sigma\cos\psi} \tag{16}$$

The effect of two displaced vortex series must now be considered. Since it is of interest to examine whether or not the velocity at the point $\varphi = 180^{\circ}$ essentially contributes to the change in temperature, it suffices to choose a schematic vortex arrangement (fig. 7, bottom) wherein

$$\sigma^2 = 1 + (1 + m\omega)^2$$

$$\cos\psi = \frac{1 + m\omega}{\sigma}$$

Thus for the upper row,

$$\left(\frac{c}{\Gamma/2\pi R}\right)_{upper} = \sum_{m=0}^{\infty}\left(1 + \frac{2m\omega}{1 + m^2\omega^2}\right)$$

is obtained, and for the lower row, shifted by half a partition

$$\left(\frac{c}{\Gamma/2\pi R}\right)_{lower} = - \sum_{m=0}^{\infty}\left(1 + \frac{\omega(2m + 1)}{1 + \omega^2\left(\frac{2m + 1}{2}\right)^2}\right)$$

The resultant velocity (positive upward) is

$$\left(\frac{c}{\Gamma/2\pi R}\right) = \omega \sum_{m=0}^{\infty}\left(\frac{2m}{1 + \omega^2 m^2} - \frac{2m + 1}{1 + \omega^2\left(\frac{2m + 1}{2}\right)^2}\right) \tag{17}$$

Since ω is rather large (for a Kármán series, ω is approximately 8), the expression under the summation sign can be simplified. Found approximately is the following equation:

$$\frac{c}{\Gamma/2\pi R} = \omega\left\{- \frac{1}{1 + \omega^2/4} + \frac{2}{\omega^2}\sum_{m=1}^{\infty}\frac{1}{m(2m + 1)}\right\} \tag{18}$$

The calculation yields, for $\omega = 8$, the value -0.32. Since in the calculation

$$n\Gamma = 1.17\,\frac{U^2}{2}$$

$$n = 0.2\,\frac{U}{2R}$$

was taken, there results as the velocity magnitude at the rearmost point for the given vortex position

$$c = 0.30U$$

It must be considered that c is periodic with the mean value of zero. Therefore, for c^2, for instance,

$$\overline{c^2} = 0.10\,\frac{U^2}{2}$$

The related temperature drop due to incomplete recovery is to be regarded as a small fraction of the nonsteady contribution.

APPENDIX 3

TEMPERATURE EFFECTS ON THE APPROACH OF A PAIR OF VORTICES TOWARD A WALL

A simple example of a nonsteady motion which can be carried out in all details is a pair of vortices moving toward a plane wall. The incompressible flow was calculated by Gröbli[7] a long time ago (fig. 8).

If a and b are the coordinates of a free vortex, there follows for the velocity of the vortex cores:

$$\left.\begin{aligned} \frac{da}{dt} &= \frac{\Gamma}{4\pi}\left\{\frac{1}{b} - \frac{b}{r^2}\right\} = \frac{\Gamma}{4\pi r^2}\frac{a^2}{b} \\ \frac{db}{dt} &= -\frac{\Gamma}{4\pi r^2}\frac{b^2}{a} \end{aligned}\right\} \tag{20}$$

Hence,

$$\frac{da}{db} = -\frac{a^3}{b^3} \tag{21}$$

The solution is

$$r = \frac{ab}{s} \quad \text{or} \quad r = \frac{2s}{\sin 2\theta}.$$

With $r_{min} = 2s$ there follows:

$$r = \frac{r_{min}}{\sin 2\theta} \tag{22}$$

The potential at the point 0 is given by

$$\phi_0 = \frac{\Gamma}{2\pi} 4\theta = \frac{2\Gamma}{\pi}\theta \tag{23}$$

The energy at the point 0 is

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[7] Gröbli, W.: Die Bewegung paralleler geradliniger Wirbelfäden. (Zürich), 1877.

$$\epsilon_0 = \epsilon_\infty - \frac{\partial \phi}{\partial t} = \epsilon_\infty - \frac{2\Gamma}{\pi} \frac{\partial \theta}{\partial t}$$

since $\theta = \text{arc tan } b/a$,

$$\frac{\partial \theta}{\partial t} = \frac{1}{r^2}\left\{a \frac{db}{dt} - b \frac{da}{dt}\right\} = -\frac{\Gamma}{4\pi r^2}$$

Thus,

$$\epsilon_0 = \epsilon_\infty + \frac{\Gamma^2}{2\pi^2 r^2} \tag{24}$$

Since a stagnation point lies at the zero point, there is also

$$T_0 = T_\infty + \frac{\Gamma^2}{2\pi^2 r^2 c_p} \tag{24a}$$

At a somewhat larger distance the vortex velocity is

$$= \frac{\Gamma}{4\pi s} = w_\infty$$

Thus, there is obtained

$$T_0 - T_\infty = \Delta T = \frac{w_\infty^2}{c_p} 8 \frac{s^2}{r^2} \tag{25}$$

For the maximum case

$$\Delta T_{max} = 2 \frac{w_\infty^2}{c_p} \tag{25a}$$

For an approach velocity of, for instance, 100 m/sec, a maximum temperature rise of approximately 20° would thus occur.

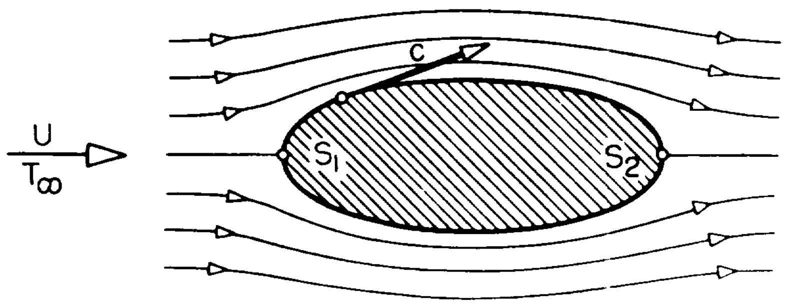


Figure 1. - Flow without separation about a cylindrical body.

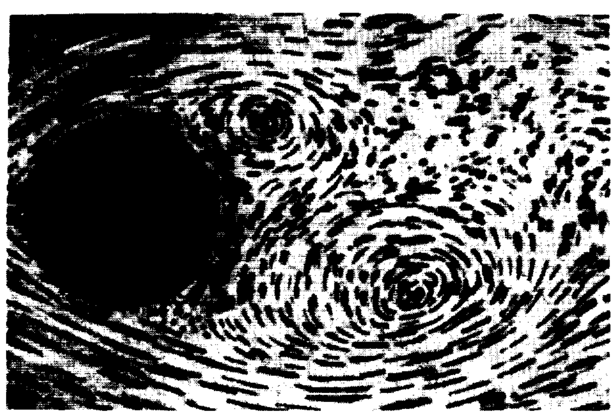


Figure 2. - Regulated flow directly at the cylinder. During the test the cylinder was flexibly mounted in the transverse direction with small amplitude.

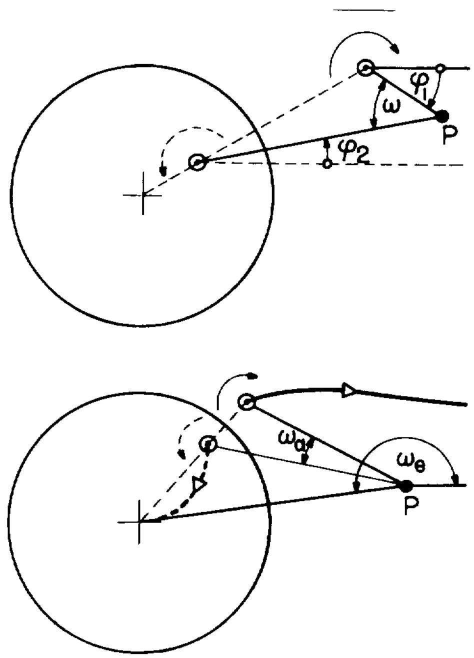


Figure 3.- Variation of potential in the flowing away of a vortex.

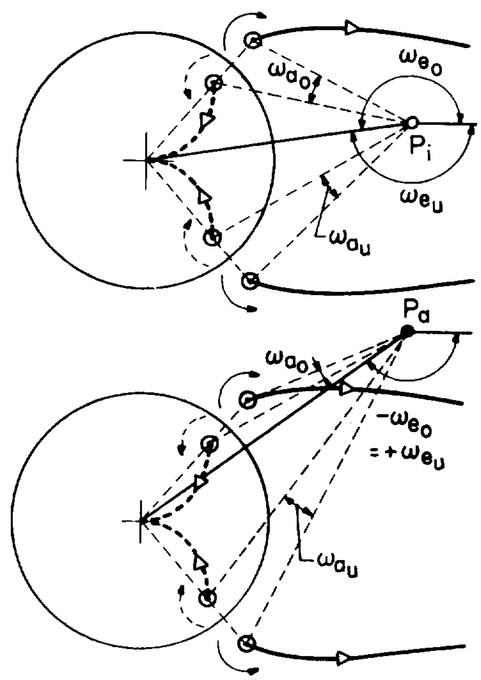


Figure 4.- Different behavior at characteristic points inside and outside the wake.

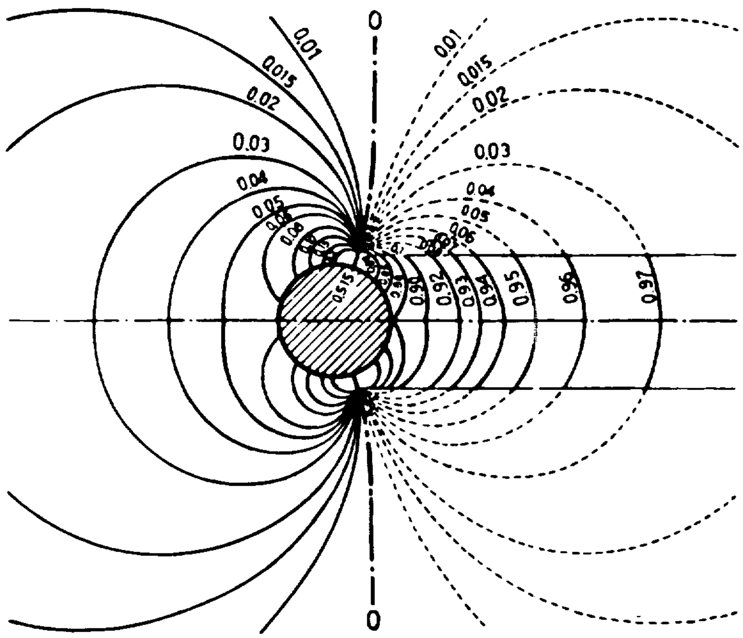
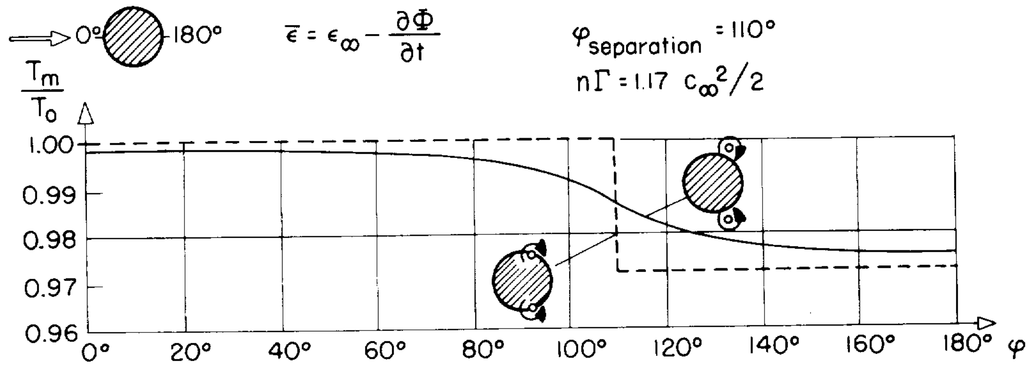
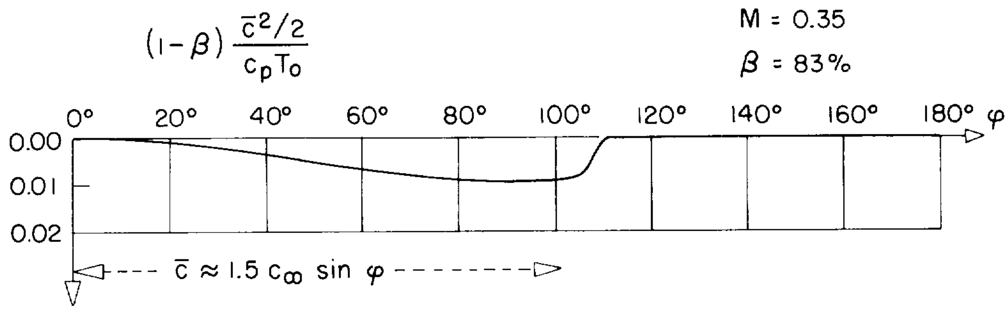


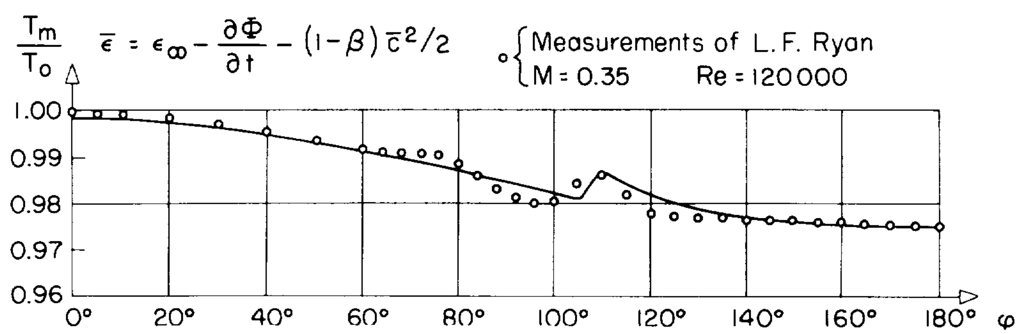
Figure 5. - Temperature changes in the flow field stemming from the periodic departure of n pairs of vortices per second, with a circulation of $\pm\Gamma$. (Solid curves: cooling; dashed curves: heating.)



Numerical temperature variation for the Mach number 0.35 and two different starting points of the vortices (nonsteady contribution).



Temperature drop due to incomplete recovery.



Comparison of the calculated temperatures and of the temperatures measured on the cylinder by Ryan.

Figure 6.

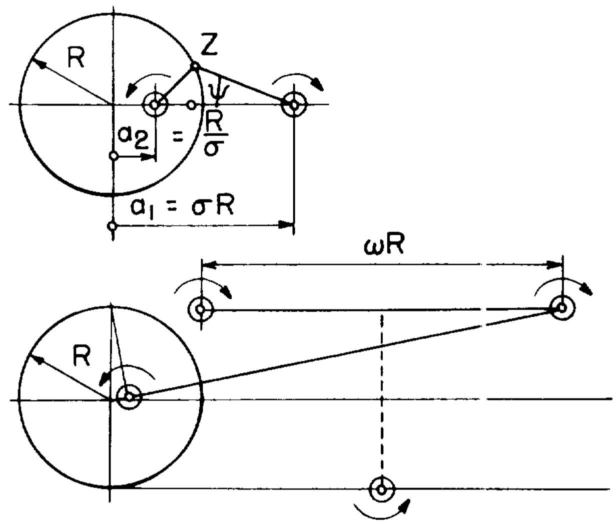


Figure 7. - Concerning calculation of the velocities at the cylinder.

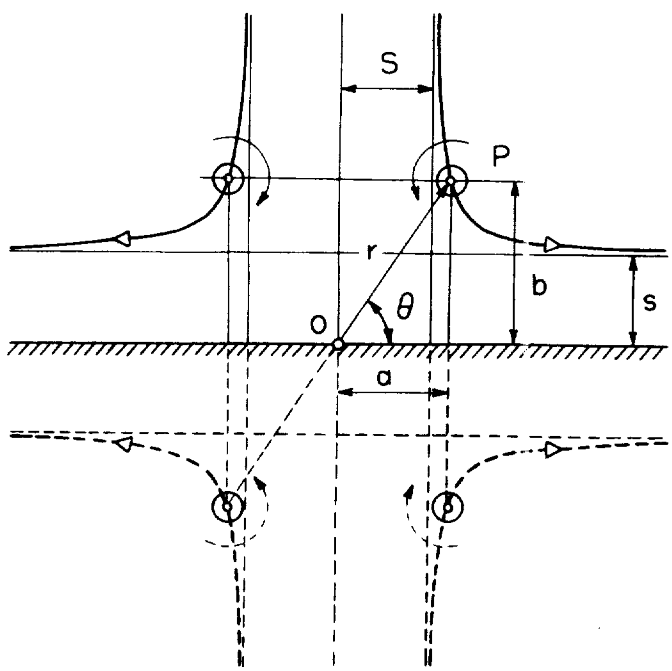


Figure 8. - Path of a pair of vortices (two-dimensional problem) in the neighborhood of a plane wall; pairs of vortices and reflected vortices.